

## Discrete Mathematics: Home-work 2

Nilanjan Datta

IAI, TCG CREST

1. Suppose that Hilbert's Grand Hotel is fully occupied, but the hotel closes all the even numbered rooms for maintenance. Show that all guests can remain in the hotel.
2. Inspector Bob has to put some criminals in cells of the prison. The criminals are notorious and can beat one another to death. If any criminal dies inside the cell, then the inspector will lose his job. In this scenario, the inspector thought of putting each criminal in a cell. But, his boss wants it to be done using the minimum number of cells. The only saving grace for the inspector is that the criminals fight according to the following pattern: (i) a criminal does not beat himself, (ii) if criminal  $C_1$  does not beat criminal  $C_2$ , then criminal  $C_2$  also does not beat criminal  $C_1$ , (iii) if a criminal  $C_1$  does not beat a criminal  $C_2$ , and criminal  $C_2$  does not beat criminal  $C_3$ , then criminal  $C_1$  does not beat criminal  $C_3$  and vice-versa. Help the inspector by solving this problem efficiently.
3. Consider an  $n \times n$  chess board where you can place some queens following the rule that if you place a queen in  $i^{th}$  row,  $j^{th}$  column then you can't put a queen in the  $i^{th}$  column,  $j^{th}$  row, where  $i \neq j$ . How many possible board configurations can you make?
4. Prove that if  $n^2 + 1$  points are placed in an equilateral triangle (the region inside as well as the perimeter) of side length 1, then there are two points whose distance is at most  $\frac{1}{n}$ .
5. Find the number of non-negative integers less than  $N$  that are relatively prime to  $N$ .
6. Prove there is a number consisting entirely of ones (e.g. 1, 11, 111, ...) that is divisible by 7777.
7. In how many ways can you distribute  $k$  identical pieces of candy to  $n$  children such that each child gets a candy?