

Discrete Mathematics: Home-work 1

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1. **Byomkesh and the Liar:** Byomkesh has interviewed four witnesses to a crime. From the stories of the witnesses Byomkesh has concluded that if Sucheta is telling the truth then so is Saikat; Saikat and Sudipa cannot both be telling the truth; Sudipa and Shreya are not both lying; and if Shreya is telling the truth then Saikat is lying. For each of the four witnesses, can Byomkesh determine whether that person is telling the truth or lying?
2. **Glums or Plogs:** There are two tribes of trolls, the Glums and the Plogs. Glums are all truth-tellers and Plogs are all liars. You meet two trolls one day, Kim and Coin. Kim says "*We are from different clans.*" Coin says "*Kim is a liar.*" Which tribe is Kim from and which tribe is Coin from?
3. **Identifying the Heavier Coin:** Prove that, if exactly one coin in a group of 3^n coins is heavier than the rest, that coin can be found using only n weighings on a balance.
4. **Sum Game:** Sourav and Sagnik alternately choose numbers from among $1, 2, \dots, 9$, without replacement. The first to obtain 3 numbers which sum to 15 wins. Does Sourav (the first to play) have a winning strategy?
5. **Spread of Corona:** Corona spreads among the squares of an $n \times n$ checkerboard in the following manner: If a square has two or more infected neighbors (those which share a common side), then it becomes infected itself. What is the minimum number of infected squares required to infect the whole board eventually. Prove that this is indeed the minimum.
6. **Predict the Hat:** 100 prisoners are lined up and assigned a random hat, either red or blue. There can be any number of red hats. Each prisoner can see the hats in front of him but not behind. Starting with the prisoner in the back of the line and moving forward, they must each, in turn, say only one word which must be "red" or "blue". If the word matches their hat color they are released, if not, they are killed on the spot. They can hear each others answers, no matter how far they are on the line. A friendly guard warns them of this test one hour beforehand and tells them that they can formulate a plan where by following the stated rules, 99 of the 100 prisoners will definitely survive, and 1 has a 50/50 chance of survival. What is the plan to achieve the goal?