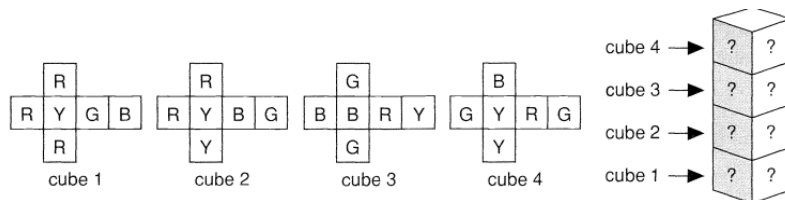


# Assignment 3

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1. There are 10 teams in a IPL, where every team plays with each of the other teams exactly once. In each round the teams are paired and they play accordingly. Prove that after 4 rounds, there are three teams who have not played against each other.
2. In a graph  $G$  with 10 vertices, among any three vertices of  $G$ , at least two are adjacent. Find the least number of edges that  $G$  can have. Can you draw such a graph?
3. Eve thought of a positive integer that is below 100 and is divisible by 7. In addition to the public knowledge above, Eve privately told the units digit of her number to Alice and she told the tens digit to Bob. Alice and Bob are very logical people, but their conversation might seem strange:  
 Alice: You do not know Eve's number.  
 Bob: Now I know Eve's number.  
 Can you deduce the number from this conversation?
4. Given four cubes whose faces are coloured red, blue, green and yellow, as in the figure below, can we pile them up so that all four colours appear on each side of the resulting  $4 \times 1$  stack?



5. Prove that a  $k$ -regular graph with girth 5 has at least  $k^2 + 1$  vertices.
6. Robot Sophia is walking on a cyclic track. The track is marked at evenly spaced intervals with 0s and 1s, with a total of 16 marks. Sophia can see the 4 marks closest to her. How should the 0s and 1s be put on the track so that she knows where on the track she is by just looking at the 4 closest marks?